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THEORETICAL MODEL FOR THE THERMAL EMISSION MEMORY EFFECT IN ROCKS

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A model for the thermal emission memory effect in rocks under cyclic heating with the temperature amplitude increasing from cycle to cycle is validated. The model is used to consider one of the possible mechanisms of the effect related to the temperature gradient on the faces of cracks dividing structural elements of a geomaterial.

Key words: *theoretical model, rock memory, cyclic heating, acoustic emission, mathematical modelling.*

The thermal emission memory effect, which is manifested in cyclic heating of rocks with the temperature amplitude increasing from cycle to cycle, involves irreproducibility of acoustic emission before the attainment of the maximum temperature of the previous cycle and a sudden increase in the acoustic emission activity and the total acoustic emission upon the attainment of the indicated temperature [1].

The thermal emission effect was discovered in the 1970es in studies of Westerly granite [2]; later, this effect has been found in rocks of various origin and composition, such a marble, potassium salt, basalt, anhydrite, quartz, etc. Results of experiments have shown that, for each type of rock, there is a characteristic range of temperatures and the maximum rate of temperature rise at which this effect is the most pronounced. However, for all rocks, the clarity of the effect in the $(i+1)$th cycle increases as the time of exposure of the samples to the maximum temperature of the ith cycle $T^i_{\rm max}$ is increased and as the time interval between successive heating cycles is decreased [3, 4].

It should be noted that there have been a few experimental studies of the thermal emission memory effect because of difficulties in measurements of acoustic emission parameters in heated rock samples. Attempts to develop a model of the effect considered have not been undertaken. However, the development of such models is necessary for both a correct interpretation of results of thermal emission measurements and for solving research problems of the physics of strength, plasticity, and thermal failure of geomaterials.

The thermal emission memory effect manifests itself as the Kaiser effect provided that mechanical effect on the subject of research is replaced by thermal effect [3]. However, the first of these effects is described by tensor quantities, and the second by scalar quantities, which prevents the use of the Kaiser effect model [5] to explain the nature and mechanism of the thermal emission memory of rocks. The purpose of the present paper is to develop a model for the thermal emission memory effect based on one of the possible mechanisms of formation and manifestation of the acoustic emission memory of rocks about the maximum thermal effects they experienced previously.

To solve the problem considered, we make the following assumptions.

1. The rock volume is represented by a set of structural elements with different thermal properties (in particular, different thermal conductivities).

2. The structural elements may be mineral grains, their aggregates or constituents of these aggregates that are not necessarily grains.

3. On the boundaries between the structural elements there are microcracks of the characteristic size $2L$, and the volume considered is exposed to instantaneous heating along the boundaries to a given temperature.

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Subsequently, the thermal energy thus delivered is distributed among the structural elements according to their thermal properties.

4. The crack faces can close neither before nor after the thermal effect, and friction is absent on the crack faces during crack growth.

5. The crack opening is very insignificant, and they have little effect on the thermal field in the rock volume considered. The boundaries of the structural elements have a greater influence on the thermal field than the microcracks located along the boundaries.

In view of the adopted assumptions, the heating of the sample can be modeled by changing the maximum temperature of instantaneous heating along the boundaries and ignoring time factors (the temperature rise rate, the exposure time between the cycles).

Each of the heating-induced acts of crack initiation or growth is accompanied by a unit act of acoustic emission, which, in view of its origin, can be called thermoacoustic emission. In this case, according to the concepts of fracture mechanics, crack growth occurs if the stress intensity factor K exceeds a certain critical value K_c.

Let a homogeneous thermal flux of constant intensity q acts on a quasihomogeneous isotropic medium which contains a crack of size $2L$ and opening s which is perpendicular to the flux direction. Using the solution of the thermal elasticity problem [6], for cracks of the first, third, and second types, respectively, we obtain the following expressions for the stress intensity factors:

$$K_{\rm I}(\pm L) = K_{\rm III}(\pm L) = 0, \qquad K_{\rm II}(\pm L) = \mp \frac{\alpha E \sqrt{\pi}}{4(1-\nu)\lambda}\, q L^{3/2}. \tag{1}$$

Here α is the linear thermal-expansion coefficient, E is Young's modulus, ν is Poisson's ratio, and λ is the thermal conductivity.

Since $q/\lambda = \Delta t/s$ (Δt is the temperature gradient on the crack faces), the second relation in (1) can be written as

$$K_{\rm II}(\pm L) = \mp \frac{\alpha E \sqrt{\pi}}{4(1-\nu)s}\, \Delta t L^{3/2}. \tag{2}$$

In the two-dimensional formulation of the problem, crack growth under the action of a temperature field occurs if the stress intensity factor of the second type (2) exceeds the critical value K_c:

$$K_{\rm II}(\pm L) \geqslant K_c. \tag{3}$$

In the three-dimensional case, it is necessary to consider a disk-shaped crack of radius L, on whose surfaces the thermal fluxes have different directions. In view of the aforesaid, the stress intensity factors of the second, third, and first types near at the crack tip are given by the following expressions, respectively [6]:

$$K_{\rm II}(\pm L) = K_{\rm III}(\pm L) = 0, \qquad K_{\rm I}(\pm L) = \mp \frac{\alpha E \sqrt{\pi}}{4(1-\nu)s}\, \Delta t L^{3/2}. \tag{4}$$

Thus, in the three-dimensional case, crack growth under the action of a temperature field occurs if the stress intensity factor of the first type (4) exceeds the critical value K_c:

$$K_{\rm I}(\pm L) \geqslant K_c. \tag{5}$$

The temperature fields leads to a redistribution of mechanical stresses near the crack tip, resulting in a reduction in the stress intensity factor and further crack growth. When the characteristic crack size reaches a certain critical value, the crack ceases to grow. The next stage of heating leads to further crack growth and the next act of acoustic emission, etc.

Since, in the rock volume subjected to heating, the initial microcracks can have different lengths and the crack growth is not an avalanche-like process but occurs with some smearing (Fig. 1).

For numerical modeling of the thermal emission memory effect using the model considered, it is necessary to determine the temperature gradient on the crack faces Δt. This problem can be solved only with the rock structure taken into account. We determine the value of Δt in the case where the structural elements of the rock volume have the same composition but different thermal properties (in the case of granular rock, a polycrystalline aggregate with differently oriented grains).

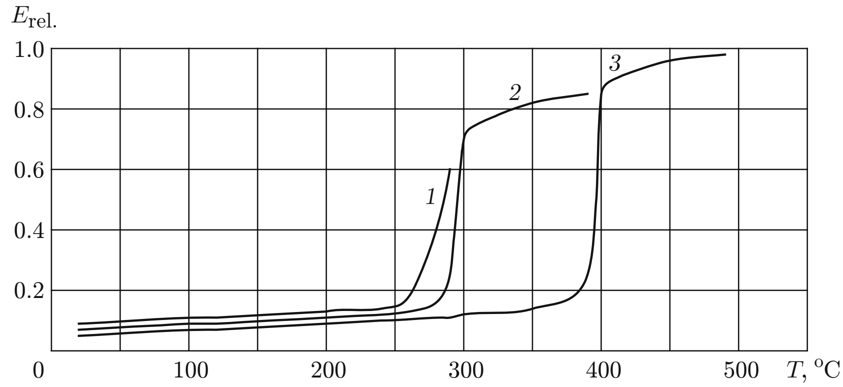

Fig. 1. Smoothed dependence of the relative acoustic-emission activity on temperature in a potassium salt sample [1]: 1) first heating cycle; 2) second heating cycle; 3) third heating cycle.

The stationary thermal field in a nonuniform infinite polycrystalline medium will be determined assuming that the medium consists of a set of finite regions with different thermal properties and for the temperature rise rate specified at infinity.

We consider the following auxiliary problem. Let the host medium contain an inclusion of finite size with the thermal conductivity different from the thermal conductivity of the medium. The indicated medium, in which the thermal conductivity tensor $\Lambda^0_{\alpha\beta}(x_n)$ is specified, is exposed to instantaneous heating to temperature T. It is required to find the temperature distribution in the inclusion, whose thermal conductivity tensor is $\Lambda^1_{\alpha\beta}(x_n)$. Summing the obtained temperature distributions in each inclusion over the entire inhomogeneous medium, which is treated as a set of the indicated inclusions, one can construct the temperature field and determine the temperature gradients on the boundaries of the inclusions.

To solve the auxiliary problem, we use the thermal conductivity equation

$$\nabla_\alpha \sigma_\alpha(x_n) = -q(x_n),$$

where $q(x_n)$ is the total heat-release rate in unit volume (in the absence of sources, it is equal to zero) and $\sigma_\alpha(x_n)$ is the thermal flux density. It is obvious that

$$\sigma_\alpha(x_n) = \Lambda_{\alpha\beta}(x_n)\varepsilon_\beta(x_n),$$

and $\varepsilon_\beta(x) = \operatorname{grad} T$. We obtain the tensor differential equation of the second order

$$\nabla_\alpha \Lambda_{\alpha\beta}(x_n)\nabla_\beta T(x_n) = -q(x_n). \tag{6}$$

Introducing Green's function $G(x_n)$, from (6) we obtain the integral equation

$$(\operatorname{grad} T)_\alpha + \int\limits_V K_{\alpha\beta}(x_n - x'_n)\Lambda'_{\beta\mu}(x'_n)(\operatorname{grad} T)_\mu \, dV = (\operatorname{grad} T^0)_\alpha, \tag{7}$$

where $K_{\alpha\beta}(x_n) = -\nabla_\alpha \nabla_\beta G(x_n)$, $\Lambda_{\beta\mu}(x_n) = \Lambda^1_{\beta\mu}(x_n) - \Lambda^0_{\beta\mu}(x_n)$, V is the volume ($V \to \infty$), and $(\operatorname{grad} T^0)_\alpha$ is the temperature gradient tensor in the ambient medium.

By virtue of the definition of Green's function, $\nabla_\alpha \Lambda^0_{\alpha\beta}(x_n)\nabla_\beta G(x_n) = -\delta(x_n)$; therefore, according to [7, 8] $G(x_n) = 1/(4\pi \boldsymbol{r}(x_n))$ in the three-dimensional formulation and $G(x_n) = \ln(\boldsymbol{r}(x_n))/(2\pi)$ in the two-dimensional formulation.

Using a Fourier transform, we calculate the integral on the left side of Eq. (7), which is equal to the difference in temperature gradient between the inclusion and the medium. As a result, we obtain

$$A = A^0_{\alpha\beta} = \frac{1}{\xi} \int\limits_V K^*_{\alpha\beta}(u;k)\, dV,$$

where ξ is a normalizing factor which depends on the dimension of the problem, V is the unit area (volume of the inclusion) over which the integration is performed, and $K^*_{\alpha\beta}(u;k) = (k_\lambda \Lambda_{\lambda\mu} k_\mu)^{-1} \cdot k_\lambda k_\mu$.

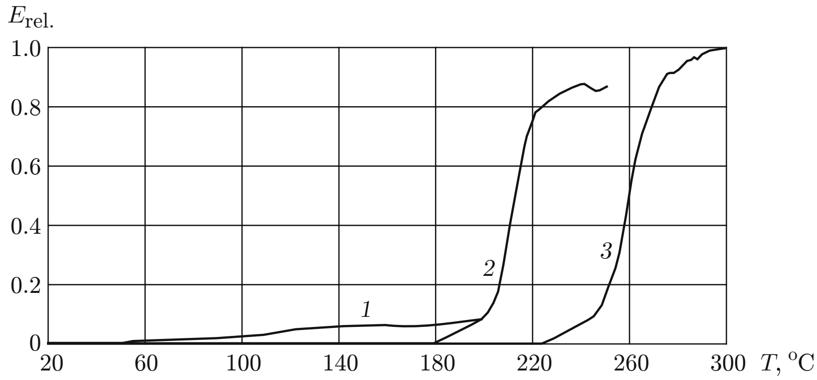

Fig. 2. Calculated relative acoustic-emission activity versus temperature (notation the same as in Fig. 1).

Using the inverse transform, we determine the required difference in temperature gradients between the boundaries of the inclusion and the medium:

$$(\operatorname{grad} T^0)_\alpha - (\operatorname{grad} T)_\alpha = (I + A(\Lambda^1 - \Lambda^0))^{-1}T \tag{8}$$

(I is the unit bivalent tensor).

To verify the above assumptions of the nature of the thermal emission memory effect, we performed numerical modeling using the computation model constructed. The inhomogeneous one-component polycrystalline medium modeling a rock sample was assumed to consist of arbitrarily oriented identical quartz grains, whose thermal conductivity tensor normalized to the principal axes was $\Lambda^0 = \operatorname{diag}(6.5, 6.5, 11.3)$ [9]. From the calculations, the average thermal conductivity tensor of this medium was $\langle \Lambda^0 \rangle = 8.1$ J/(m · sec · K). [The model included 10^6 grains ($100 \times 100 \times 100$).]

This one-component polycrystalline medium was exposed to cyclic heating to specified temperatures. For this, the temperature field in all quartz grains was calculated for temperature variation with a specified step (1°C) and the temperature gradient between neighboring grains was determined according to formula (8). Heating was performed cyclically: in the first cycle, the temperature was increased from 20 to 200°C, in the second cycle, from 20 to 250°C, and in the third cycle, from 20 to 300°C. It was assumed that, after the heating, the medium cooled to a temperature of 20°C but, for this stage, the temperature gradient on the grain boundaries were not calculated. In this one-component polycrystalline medium, arbitrarily oriented cracks 0.1–2.0 mm long were specified on the grain boundaries. It was assumed that, under the action of the temperature stresses arising near the crack leads, the crack length began to increase [if condition (5) was satisfied] or remained unchanged and that the crack opening was constant and independent of the temperature field and geometrical dimensions. The number of cracks was set equal to 10,000.

Each act of crack growth is accompanied by an act of acoustic emission; therefore, in the model, the acoustic emission activity increased during growth of one concrete crack. According to formulas (3)–(5), crack growth can proceed to infinity. In practice, however, this process decays rather rapidly; therefore, the model was supplemented with additional limitations: it was assumed that, during growth of one crack, the crack length increased by not more than 25% and that Young's modulus in the quartz grains surrounding the crack varied so that the stress intensity factor calculated for the crack of new length was smaller than the coefficient corresponding to the beginning of growth of this crack. Thus, the model took into account the stress relaxation near a crack due to its growth, and calculated values of Young's modulus in the quartz grains neighboring the grown crack were used in the calculations for the next temperature cycle.

Figure 2 gives the results of calculation of three heating cycles using the model considered. It shows the temperature dependence of the relative acoustic-emission activity $E_{\rm rel}$ (by the relative acoustic-emission activity is meant the ratio of the number of grown cracks at a given temperature to the total number of cracks in the model). From Fig. 2 it is evident that, in the first heating cycle (to $T = 200$°C), the acoustic emission activity is insignificant and, for each current temperature value, the number of grown cracks does not exceed nine. However, in the second cycle (heating to $T = 250$°C), beginning from the time of termination of the first cycle ($T = 200$°C), for each calculated value of the current temperature, the number of grown cracks increases to 100–105. In the third heating cycle (at $T = 250°C$), there is an even greater increase in the number of grown cracks — to 120 for each current temperature value.

In the present work, an attempt was undertaken to construct and validate a mathematical model for the thermal emission memory effect. This model takes into account the single factor — the temperature gradient on the crack faces. The model ignores effects due to nonuniform heating of the rock grains and the anisotropy of f temperature expansion coefficients of these grains. These factors lead to the mechanical stresses on the grain boundaries, crack growth, and the thermal emission memory effect.

The model described here assumes instantaneous heat distribution in the sample and ignores the temperature rise rate. At the same time, it is likely that, at a low heating rates, the thermal emission memory effect will not manifest itself since the thermal stresses have time to relax and the temperature gradient on the crack boundaries is rather small. At high heating rates, effects due to nonuniform expansion of rock grains will obviously occur but these effects cannot be described by the model proposed. In addition, the model considers rocks with only one type of grains. The temperature distribution in polymineral rocks requires a separate study.

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